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Incorporating physical climate risks into banks' credit risk models

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Abstract

Over the past few years, physical risks have turned from a niche domain of (re)insurers into a systemic risk factor that may have an impact through various channels on financial markets and financial institutions alike. While physical risks are not a common income-producing or even a sizeable cost-of-business risk factor for most banks, they do affect banks, mostly indirectly, through their loan and trading books. Against this backdrop, standard setting bodies and financial regulators have increasingly called on banks to recognise physical risks as an additional factor in their risk space and internalise it in their risk management policies.

A major obstacle for banks on this way, however, is the absence of generally accepted industry models of credit risk adjusted for physical risk factors. Such models are increasingly needed to account for physical risks in banks' capital requirements, loan loss provisions, pricing of loans and, eventually, derivatives to hedge this risk. This poses the question of building a bank's internal model for climate-related correction to the internal probability of default and loss given default or using third-party databases on the type of the borrower's assets, their geolocation, exposure to climate factors, statistical description of weather events and damage functions.

This paper proposes a methodology that allows in a relatively simple way the integration of physical risk component into the credit risk modelling, using an extension of the one-factor Vasicek model. The model described by the paper may be of specific interest for both banks and regulators, as it preserves important properties of models currently used while allowing for an informed mitigation of physical risk factor in credit risk. Additionally, the paper discusses further possible extensions of the credit risk model if physical risk manifests itself in more than one state.

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Section 1 – Introduction

There is a prevalent view among meteorologists and insurers that climate-related events are becoming more frequent and severe, impacting both households and corporates. For instance, over the past four decades, the number of reported hydrometeorological disasters has surged nearly fivefold, rising from approximately 750 incidents between 1971 and 1980 to around 3,500 incidents during 2001 to 2010. Concurrently, cumulative economic losses have escalated more than fivefold, increasing from USD 156 billion to USD 864 billion per decade.[1]

For banks, climate risk manifests itself in the deterioration in the solvency of borrowers due to damage to their assets from natural disasters caused by climate change ("physical" risk) and/or due to costs of transition to a low-carbon economy caused by changes in public sector policies, legislation and regulation, changes in technology and changes in market and customer sentiment ("transition" risk). While the climate-related financial risk has not yet become a common income-producing or even a sizeable cost-of-business factor for most banks, physical risk does affect banks, mostly indirectly through their loan and trading books.

Physical risks are transmitted to a bank's balance sheet and P&L through different channels. For example, natural disasters, such as floods, hurricanes, droughts, affect households and corporate borrowers by impairing their fixed assets (housing, inventory, property, equipment or infrastructure). This, in turn, impacts the creditworthiness of the obligors by decreasing the value of collateral posted at the bank (if any) and, as a second-order effect, when damaged rental properties and factories generate less income thus undermining the obligor's creditworthiness. Since climate-driven events commonly affect multiple borrowers at the same time, thus driving up correlation of defaults in the loan portfolio, a bank may incur in significant losses if it has a concentration of credit exposures to obligors located in a high-risk region.

Against this backdrop, standard setting bodies and financial regulators have increasingly urged banks to recognise physical climate risk as a new factor in their risk space and internalise it in their credit risk management policies. Several national financial authorities have explicitly outlined supervisory expectations for banks to adjust their risk management practices in response to climate change. These adjustments typically involve disclosing and managing exposure to physical risks, as well as allocating capital accordingly. However, financial authorities acknowledge the challenges posed by the lack of data and risk-modelling capabilities.

For example, the Basel Committee on Banking Supervision (BCBS) in its "Principles for the effective management and supervision of climate-related financial risks" underscores that banks should understand the impact of climate-related risk drivers on their credit risk profiles and ensure that credit risk management systems and processes consider material climate-related financial risks. Moreover, "banks should have clearly articulated credit policies and processes to address material climate-related credit risks. This includes prudent policies and processes to identify, measure, evaluate, monitor, report and control or mitigate the impacts of material climate-related risk drivers on their credit risk exposures (including counterparty credit risk) on a timely basis."[2]

[1] World Meteorological Organization (2015).

[2] BCBS (2022a). More specific guidance on how climate-related financial risks may be captured in existing Pillar 1 standards was provided in BCBS (2022b), "To the extent that the risk profile of a counterparty is affected by climate-related financial risks, banks should give proper consideration to the climate-related financial risks as part of the counterparty due diligence.

In the European Union, banks are expected to meet supervisory expectations regarding climate and environmental risks, including their full integration into the Internal Capital Adequacy Assessment Process (ICAAP) and stress testing.[3] That said, the European Central Bank's thematic review [4], which examined banks' risk strategies and risk management processes, highlighted that the industry still lacks sophisticated methodologies and detailed information on climate and environmental risks.

In the United Kingdom, the Bank of England (BoE) highlighted emerging evidence of potential gaps in the banking capital framework and has started examining climate-related deficiencies in the measurement of risk-weighted assets. BoE acknowledges both the limitations in current modelling techniques used by banks to fully incorporate and estimate the impact of climate factors on their counterparties, as well as certain "regime gaps." These regime gaps refer to challenges in capturing climate risks due to the design or use of methodologies within the capital frameworks. Specifically, for internal ratings-based (IRB) models, BoE concludes that climate risks could potentially be reflected through the modelling of risks to exposures, although data and modelling challenges are likely to persist.[5]

Finally, the Group of Central Bank Governors and Heads of Supervision (GHOS), the oversight body of the Basel Committee on Banking Supervision, has recently agreed to prioritise further analysis on the financial risk implications of extreme weather events and tasked the Basel Committee to analyse the impact of such events on financial risks[6].

Apart from the challenge of forecasting low-frequency-high-severity climate events, in which banks are clearly not specialists (unlike property insurers), estimating economic loss to their borrowers induced by such events is another major problem for banks. Loss models developed in the insurance industry may be only partially useful, as they only transpose physical impact to impairment of fixed assets. However, banks need to have also the second type of models estimating how this impairment could affect the creditworthiness of the borrower. Given the dearth of relevant statistical data, such models are scarcer than those for transition risk, are often based on econometric estimates of stressed probabilities of default derived from the firm's financials under equally likely stress scenarios (ECB 2022a) rely on a widely diverse setup and assumptions. Meanwhile, practical models of credit risk adjusted for physical risk are increasingly needed to account for physical risk in pricing of loans banks', loan loss provisions, capital requirements, and, eventually, hedging this risk with derivatives.

In supervisory climate stress-testing exercises, physical risks are often considered marginal or moderate in the short term. These risks are supposed to materialize as an outcome of adverse long-term scenarios of climate change (i.e. temperature rise) in the absence of adequate policy action. Furthermore, banks may assume that the impact of higher physical risk will be mitigated by insurance (BoE 2022). Even in countries with higher natural disaster risk, banks often include only physical damages in estimating the devaluation of properties (HKMA 2021) with rare examples of cascading stress scenarios down to probabilities of default and losses given default (Adams-Kane et al, 2024). Therefore, physical risk estimates tend to be too uncertain to be integrated into bank risk management framework. Unsurprisingly, bank supervisors expect banks to be capable of modelling the impact of physical risk on banks' counterparties with varying degree of granularity without prescribing though any particular model type.

To that end, banks should integrate climate-related financial risks either in their own credit risk assessment or when performing due diligence on external ratings."

[3] ECB (2020).

[4] ECB (2022b).

[5] Bank of England (2023), para 6 and 40–48.

[6] BIS (2025).

In this paper, an innovative approach for integration of physical risks to credit risk modelling is proposed. The model design and choice of its parameters accommodate both a directional climate change (i.e. climate risk per se) and weather risk due to increased volatility of 'climate' risk factors.

The seminal Vasicek model (2002)[7] which underlies many internal credit risk models at banks, as well as the regulatory internal ratings-based (IRB) approach in Basel II and Basel III, is considered as a starting point. Physical risk is then modelled as a single stochastic factor that manifests itself in a binary way with an externally defined probability and a jump in the market value of assets of an individual corporate borrower.

Next, a portfolio model is developed to explicitly measure the contribution of physical risk to credit risk losses. Besides common purposes, such as economic capital allocation for unexpected credit losses and loan loss provisioning for expected losses, the proposed model may also be instrumental for banks in managing this new factor of credit risk by hedging it with derivatives on climate-induced damage.

The research demonstrates that the proposed credit risk model enhanced by the physical risk factor preserves the so-called portfolio invariance property, i.e. the invariance of the risk measure for a single credit claim to the composition of the loan portfolio to which it is added. This important property is highly desirable from a practical viewpoint to avoid time-consuming full re-calculation of a risk measure on a portfolio level, as well as for its prospective suitability for regulatory purposes.

The paper is structured as follows. Section 2 provides a review of the recent literature, focusing on current methods for integrating climate risk into credit risk measurement and modelling for banks. It is shown that most of the models describe the default process of an individual corporate borrower but do not consider a case of a loan or a bond portfolio. Section 3 examines the main methodological issues for integrating physical climate risk into credit risk modelling and suggests possible ways to move forward. A parsimonious approach to integrating climate risk into credit risk assessment on a portfolio level is then developed in Section 4, and Section 5 is dedicated to further extensions of the proposed credit risk model. These include a multi-state distribution approach (i.e. if physical risk related to climate events manifests in more than one state), extending the model for transition risk, possible regulatory applications, and hedging physical risk with climate damage index swaps. Appendix 1 gives an illustrative example which demonstrates how meaningful the adjustments are in the current environment.

Section 2 – Literature review

To date, there has been relatively little research on the methodology for incorporating physical risks into credit risk measurement and management that could be applicable in the banking sector. However, it should be noted that the number of publications on this topic has grown significantly over the past few years. Such work has often turned to empirical studies of the contribution of climate factors to the credit risk of securities traded in financial markets focusing on transition risk and much less on physical risk.[8] Theoretical models in this field are still scarce.

Part of the research in this area focuses on the so-called structural models of credit risk. In these models, default occurs when the value of a company's assets falls below a certain threshold level relative to its liabilities and the change in the asset value over time is governed by some stochastic process.[9] More specifically, this line of work aims to integrate climate risk into the original Merton model of corporate

[7] Vasicek (2002).

[8] See, for example, the review by Eren, Merten, and Verhoeven (2022), Bressan et al (2024).

[9] Blasberg and Kiesel (2024). The usability of this type of models for embedding climate risk into credit risk was earlier noted by Monnin (2018).

default and its most used modifications. In particular, in these models, transition risk is described as an adjustment to the (constant) growth rate of assets modelled by a continuous stochastic process[10] or as a discrete impact (jump) in the asset value.[11]

A similar approach may be used for modelling physical risk. For example, Agliardi and Agliardi (2021) and Kölbel et al. (2022) model the scenarios for the impact of physical risk (the frequency and severity of the shock) on a company's EBITDA and, consequently, on its asset value. This scenario-based method is further developed by Le Guenedal and Tankov (2024) who employ a Bayesian approach to account for uncertainty in scenarios that differ in the frequency of climate risk-driven events, but not in their magnitude.

Bell and Vuuren (2022) model climate risk as a stochastic impact on asset value in the Merton model's setting. In their study, only physical risk leads to an increase in the volatility of the firm's asset value, while transition risk has been anticipated and already accounted for in the stock price of the corporate obligor and, consequently, in the volatility of its asset value. The authors use stochastic simulation to generate the climate risk-adjusted future value of the company's assets and its volatility which are then fed as inputs to Moody's CEDF model (2023) to calculate climate risk-adjusted default rates.

Moody's Climate-Adjusted EDF (CEDF) Model [12]

CEDF[13] is a structural credit risk model that calculates the probability of default over a horizon ranging from one to 30 years for about 40,000 companies, considering both physical and transition risks. The range of modelled physical risks is broad, covering "acute" events (such as hurricanes, wildfires, and floods), as well as slow "chronic" processes, including sea level rise, heat stress, and water stress. The Moody's model is built on an array of NGFS I and II scenarios of global warming, scenarios developed by the Monetary Authority of Singapore, as well as the Shared Socioeconomic Pathways developed by the IPCC.

The CEDF model of physical risk is constructed in a top-down manner, starting from the estimation of climate change damages at the global economy level down to adjustments to the parameters of the stochastic asset value path of a specific corporate borrower. The process involves a number of steps:

First, high-level models, such as integrated assessment model (IAS) and aggregate damage functions, are used to estimate the climate-driven damage as a percentage of global GDP under various future temperature change scenarios.

Second, this measure of damage is projected to the locations of a corporate borrower's assets that are most exposed to physical risk. This mapping is done using Moody's ESG (MESG) scores for specific locations of the firm's assets, as well as external data sources. MESG scores are a composite of the firm's operations risk, supply chain risk, and market risk with weights 70%, 15%, and 15%, respectively.

Third, the frequency of climate events that could cause the estimated magnitude of damage is derived from the damage projection for the specific location. Physical risk in this model is described through the change in the average frequency of weather events caused by climate change.

Fourth, the model estimates how such events have historically impacted companies' earning and asset values based on published empirical research.

Finally, the model derives, for each firm, the change of the mean and variance of the normal probability distribution describing the time evolution of the company's asset value to arrive at a climate risk-adjusted expected default frequency.

[10] Blasberg, Kiesel, and Taschini (2022).

[11] Bouchet and Le Guenedal (2022).

[12] Moody's (2023).

[13] CEDF is a further development of the EDF model for public companies originally developed by KMV (1991).

Similarly, Hahn (2022) proposes to use a Merton-type credit risk model to estimate the probability of the obligor's default and its credit rating migration that reflect climate risk factors. In order to do so, the author leverages multilevel covariance analysis to estimate the impact of climate factors on the firm's asset values depending on macroeconomic drivers. In this model, the realised climate risk affects firms through different transmission channels: e.g. rising temperatures hit agricultural production and, down the chain, GDP, while hurricanes raise home insurance premiums which affects the prices of residential real estate. Hahn proposes to use linear regression to estimate the covariance of log-changes of GDP and the residential real estate price index with the normalised changes in climate factors.

The above models describe the default process of an individual corporate borrower but do not consider a case of a loan or a bond portfolio. Against this background, Bourgey, Gobet, and Jiao (2024) build a comprehensive set of models for a large bond portfolio subject to both transitional and physical risks. The modelling framework is "top-down" in that it starts from the Shared Socioeconomic Pathways describing climate change scenarios brought to the level of a single corporate borrower, but it is also "bottom-up" to the extent that it accumulates obligor's losses at the loan portfolio level. Physical risk in this work is affected by Nordhaus' DICE integrated assessment model [14]and is hence driven by the results of the borrower's adaptation to climate change. The authors calibrate a stochastic jump-diffusion process to estimate physical risk at the obligor level, and account for correlations between borrowers' defaults in the portfolio through their correlations with a systematic risk factor. Finally, to reduce the dimensionality of the computational problem at the top-down portfolio level the authors develop a numerical method based on the principal component analysis and polynomial chaos expansion.

In summary, the following conclusions can be drawn from the studied literature. First, current research has predominantly focused on transition risks rather than physical risks. The reviewed papers do not integrate both physical and transition risk within a single model. Second, the probability of default adjusted for climate risk is primarily calculated using structural default models, which often model default for individual borrowers. Only a few credit portfolio models consider both default correlations and climate risk factors. Lastly, most of the models developed to date do not appear to be well-suited for practical use in banking credit risk management.

Section 3 – Methodological challenges in integrating physical risks into credit risk modelling

Banks using internal models for credit risk may be able to incorporate climate-related factors into their credit risk assessments. However, integrating climate-related risks into credit risk modelling present methodological challenges.

Credit risk in banking is measured and managed by splitting it into two uneven components: expected credit loss (EL) corresponding to the long-term average of losses, and unexpected credit loss (UL) occurring due to the volatility of losses over and above its average.

Banks are expected to cover expected losses through pricing with loan loss provisions and unexpected losses – with their regulatory and economic capital. Numerous models have been developed in the industry to calculate the unexpected loss component for the loan and bond portfolios, with the Vasicek model (1987, 2002) being one of the most seminal and used both by banks and their regulators.

The asymptotic single risk factor (ASRF) model developed by Vasicek for an infinitely granular loan portfolio based on the seminal Merton model for a single obligation is defined as follows:

[14] DICE (the Dynamic Integrated Climate-Economy) model is an integrated assessment model developed by William Nordhaus. It integrates in the neoclassical economics, carbon cycle, climate science, and estimated impacts allowing the weighing of subjectively guessed costs and subjectively guessed benefits of taking steps to slow climate change.

$$DR = \left(N\left(\frac{N^{-1}(PD)+\sqrt{\rho}\cdot N^{-1}(\alpha)}{\sqrt{1-\rho}}\right) - P_D\right)$$

where

DR is the share of obligors in default in a loan portfolio;

N(x) is the standard normal distribution function, $N^{-1}(x)$ is the inverse standard normal distribution function,

P_D is a default probability of a corporate borrower,

α is a confidence probability (level) measured in decimals;

ρ is the correlation with systematic risk factor.

At first glance, using this mode as a starting point, physical risk could be accounted for in a number of ways, which include at least the following:

i) In a (higher) confidence probability of losses due to credit risk (in the IRB approach it is set at 99.9 percent, which means the probability of losses due to credit risk exceeding the bank's loss provisions and capital on average once every 1000 years),

ii) In the probability of default (P_D) of a single obligor;

iii) Through the correlation parameter with systematic (market) risk;[15] or

iv) As a new independent systemic risk factor.

The first option appears arbitrary and would simply increase regulatory capital without clear way to either measuring or mitigating climate risk.

The second option would involve estimating a P_D based on the projected (modeled) damage (to the borrower's assets) from realized physical risk as reflected in the company's financial statements and key ratios (such as EBITDA, interest coverage ratio). Banks using the IRB approach calibrate their P_Ds to be close to the so-called 'central tendency' (i.e. long-term average of default rates).[16] In this regard, since a major one-off loss from natural disasters (including those caused by climate change) is a relatively rare event (compared to a 'normal' default), it should logically not have a noticeable impact on the average default rate and hence the one-year P_D. More importantly, a forward-looking PD can hardly be estimated and verified using historical (statistical) data over a meaningfully long period of observations in the past.

This leads to a conclusion that physical risks should be reflected in the 'core' of the above formula, ie in the argument of the cumulative normal distribution function that converts the probability of borrower default into a conditional probability corresponding to unexpected losses due to credit risk. To this end, we must look in more detail to the options iii and iv above, which involve revising the systematic risk factor in the model.

There are at least two alternative ways for implementing this idea:

- specifying a physical risk component in the systematic risk factor that is already implicitly

[15] This possibility is explored by Baranović, I. et al. and PRA (2021).

[16] Basel II and III require that one-year PD should not be less than the average of observed yearly default rates calculated over at least one full credit cycle.

present in the model; or

- introducing a physical risk factor as an independent new systematic risk factor.

When considering these options, one could argue that climate risk is a factor already present to some extent in the existing systematic market risk that determines the value of the borrower company's assets. In the Vasicek model, the factor of systematic (market) risk is implicitly present and manifests itself through its (exogenously defined) correlation with the market value of the borrower's assets. The IRB approach goes even further in modelling the correlation as inversely and nonlinearly depending on the Probability of the borrower's default:

$$\rho = 0.12 \cdot \left(\frac{1-e^{-50PD}}{1-e^{-50}}\right) + 0.24\left(1 - \frac{1-e^{-50PD}}{1-e^{-50}}\right).$$

Isolating a climate risk factor would require moving from a one-factor to a two-factor model, ie splitting the single correlation with the only systematic risk factor in the model into two correlations with some weights, i.e. climate risk and residual systematic risk (as shown, for example, by Gürtler, Hibbeln, and Vöhringer (2007) for credit concentration risk).

For the IRB approach, the main objection to the above idea is the fact that the correlation function in the above formula depends inversely on the average default probability. In other words, if the borrower has a short distance to default, it is assumed to be caused mainly by some borrower-specific reasons rather than by the market as a whole. Clearly, a corporate borrower's exposure to physical risk is not correlated with the probability of default driven by systematic "economic" risk, but is due to other factors, such as its industry, the location of its fixed assets and their physical and structural characteristics, the role of specific fixed assets in the production process, and natural hazards at their location. In sum, a correlation of credit risk with the physical risk factor is hard to specify explicitly or implicitly in the present Basel correlation function for systematic economic risk, because it depends on the probability of losses due to credit risk. Therefore, the option of introducing a new independent variable in the Vasicek model to account for a correlation of the assets value with the implicit physical risk factor should be explored. Note that the correlation between physical risk and non-climate systematic risk can be assumed to be zero. This can be empirically observed, for instance, in low correlations between the yields of CAT-bonds and market risk factors (interest rates, stock indices, exchange rates and commodity contract prices).

In an ideal world, a desirable property of the credit risk model this approach is the so-called *portfolio invariance*. This property states that unexpected credit loss for an individual claim should be invariant with regards to the composition of the loan portfolio to which this claim is added. Such invariance has been guaranteed with a single systematic risk factor driving defaults of all obligors, another necessary condition being an infinitely large degree of diversification of the loan portfolio.[17] If the portfolio invariance property is not maintained by the model, this leads to impossibility of arithmetic aggregation of capital requirements for each single claim, as, e.g. under the IRB approach.

[17] See, for example, Gordy, M. (2002): A risk-factor model foundation for ratings-based bank capital rules, Board of Governors of the Federal Reserve System.

Some preliminary conclusions for integrating climate-risk into the credit-risk modelling framework can be drawn from the above:

1. Banks using the Vasicek-type portfolio models for measuring credit risk could enhance them by incorporating physical risk as a new independent systemic risk factor of unexpected credit losses covered by the bank's capital.
2. The inverse non-linear dependence of the correlation of the value of assets with the (implicit) systematic risk factor in the modified Vasicek model adopted in the IRB approach makes it problematic to account for the physical risks in the correlation parameter.[18]
3. The credit risk model enhanced with another explicit systematic risk factor should ideally have the property of portfolio invariance of the unexpected credit loss measure (and, consequently, the capital requirement to cover it) for a single credit claim to the composition of the entire loan portfolio.

Section 4 – Enhancing the Vasicek model with physical risks

The IRB approach provides banks with a formula aiming to cover unexpected losses. This is done using the Asymptotic Single Risk Factor (ASRF) calculation, which is briefly reviewed and generalised below. This section aims to demonstrate that it is possible to include the physical risk factor in the IRB calculation without changing the principles of the existing arrangement.

The Risk-Weighted Asset (RWA) formula:

$$Model\ RWA = \left\{LGD \cdot \Phi\left(\frac{\Phi(P_D)+\sqrt{\rho}\,\Phi^{-1}(Q)}{\sqrt{1-\rho}}\right) - LGD \cdot P_D\right\} \cdot MA \cdot EAD \text{ x } 12.5 \qquad (1)$$

where LGD denotes the Loss Given Default (in %),

P_D is the assumed probability of default (in %).

ρ is the correlation coefficient between assets and the systematic factor (the single risk factor). In the IRB approach, the Basel framework sets the correlation coefficient to different values depending on the asset class. In what follows, the paper explicitly keeps Basel's correlations and treats them as given constants.

Q is the confidence interval (in capital calculations, Q is taken equal to 0.999)

EAD is Exposure at Default, i.e., the amount of money a bank will likely lose if the customer defaults. This is estimated for each client, taking into account the amount drawn and the credit conversation factor.

MA is the maturity adjustment: $MA = \frac{1+(M-2.5)b}{1-1.5b}$,

P_D and LGD are estimated for every client, while the other parameters are fixed and essentially external.

For the analysis carried out here, it is important to note that Formula (1) is essentially a scaled difference between Value at Risk (VaR) corresponding to a particular confidence level Q and the expected loss, all on the portfolio level:

$$RWA = Multiples \cdot (VaR(Q) - < EL >) \tag{2}$$

However, in the ASRF model for large portfolios of small equally distributed loans the above percentage VaR of the portfolio value is equal to the probability of default $C_V(Q)$ for a single loan conditional on the particular value of the single risk parameter (systemic risk) corresponding to the confidence level Q :

$$RWA = Multiples \cdot LGD \cdot (C_V(Q) - P_D) \tag{3}$$

It should be noted that this non-trivial result, being a consequence of a particular form of the portfolio loss distribution (due to Vasicek (1997, 2002)), remains the key to the invariance property.

In what follows, the research demonstrates that the Generalised ASRF model, which includes the physical risk factor, can also be written in the same form (3), helping to keep all major characteristics of the current IRB approach, including portfolio invariance (Gordy 2003).

In the spirit of the Basel RWA formula, and of the BCBS's guidance (2022a, 2022b), this paper presumes that banks may be able to estimate internally LGD and the corresponding probabilities of default with and without physical risks, on a client-by-client basis, as they normally do. The rest of the parameters will be external and objective and can be fixed globally or on a country level.

Vasicek portfolio model with physical risks

The model assumes there are N → ∞ identical loans[19], for each of which the Merton model of defaults holds. The Merton model states that the companies' assets are modelled by a stochastic, typically log-normal process, and the default at a particular time horizon is caused by the assets being below the loan values. The difference between the Merton and Vasicek models is that in the Vasicek model, instead of explicitly modelling liability level and process parameters, P_D is given, and the parameters are inferred from it.

When recalling the setup of the Merton model of defaults (Vasicek 1997, 2002) one should consider a single loan for a company with asset value V_t modelled as:

$$V_t = V_0 e^{\mu_t t - \frac{t}{2}(\sigma_V^2 + \beta_V^2) + (\sigma_V, \beta_V)\binom{S_t}{B_t}},$$

where

B_t, S_t are two Wiener (Brownian motion) processes (systematic factor and idiosyncratic noise, respectively), with the covariance between dS_t and dB_t equals zero

σ_V denotes the sensitivity of the company's assets to the systematic risk,

μ_t denotes the average return on the firm's assets,

β_V denotes the sensitivity of the company's assets to idiosyncratic risk.

[19] The condition of identical nature of loans can be relaxed : each borrower's obligation is negligibly small compared to the size of the portfolio.

The proposal is to model the physical risk impact by introducing a hit to the assets - a jump down with a magnitude $e^{-\alpha}$ < 1 and probability q (q < 1).[20] This results in the following generalisation of the asset process:

$$\widehat{V_t} = V_t\, e^{-\alpha}\, \xi + (1 - \xi)V_t,$$

where

ξ = (0,1) – a single jump with a probability q over a time horizon T.

Since the stochastic process V_t is geometric Brownian motion, it is irrelevant for the final value of $\widehat{V_T}$, when in time t ∈ [0, T] the jump happens.[21]

Consider $T = 1$, the probability of default on the loan L is :

$$\begin{aligned} \mathrm{P}_D = \mathrm{P}\big(\hat{V} < L\big) = \\ &= (1-q)\,\mathrm{P}\left(\mu_t - \frac{1}{2}(\sigma_V^2 + \beta_t^2) + (\sigma_V, \beta_V)\binom{S}{B} < ln\frac{L}{V_0}\right) \\ &+ q\mathrm{P}\left(\mu_t - \frac{1}{2}(\sigma_V^2 + \beta_V^2) + (\sigma_V, \beta_V)\binom{S}{B} - \alpha < ln\frac{L}{V_0}\right) \end{aligned}$$

i. e. $\mathrm{P}_D = (1-q)\,\mathrm{P}\left((\sigma_V, \beta_V)\binom{S}{B} < ln\frac{L}{V_0} - \mu_t + \frac{1}{2}(\sigma_V^2 + \beta_V^2)\right) +$

$+q\mathrm{P}\left((\sigma_V, \beta_V)\binom{S}{B} < ln\frac{L}{V_0} - \mu_t + \frac{1}{2}(\sigma_V^2 + \beta_V^2) + \alpha\right).$

Since $\frac{(\sigma_V, \beta_V)}{\sqrt{\sigma_V^2 + \beta_V^2}}\binom{S}{B}$ is a new normal random variable with a variance equal to one, it is possible to rewrite the probability of default as a sum of the two terms

$$\mathrm{P}_D = (1-q)\,\Phi(C^*) + q\Phi(C^* + \hat{\alpha}), \qquad (4)$$

where

Φ(•) is a cumulative function of the standard normal distribution,

C^* is defined as

$C^* = \frac{\ln\frac{L}{V_0} - \mu_t + \frac{1}{2}(\sigma_V^2 + \beta_V^2)}{\sqrt{\sigma_V^2 + \beta_V^2}}$,

and $\hat{\alpha}$ is defined as:

$\hat{\alpha} = \frac{\alpha}{\sqrt{\sigma_V^2 + \beta_V^2}}$.

Moreover, if one introduces a ‘q-deformed normal distribution’:

$$\Phi_{q,\alpha}(x) \equiv (1-q)\,\Phi(x) + q\Phi(x + \alpha), \qquad (5)$$

[20] In terms of this binary setup, climate risk can be described as the risk of increasing of α and/or q in the future from their current values.

[21] Therefore, in what follows the authors can think of the jump as happening at the maturity point.

alongside with a shifted normal distribution

$$\Phi_\alpha(x) = \Phi(x + \alpha),$$

the expression for the probability of default (4) is condensed to the form:

$$P_D = \Phi_{q,\hat{\alpha}}(C^*).$$

In this form, the expression of the probability of default is a simple "q-deformation" of the Vasicek expression:

$$P_D^0 = \Phi(C^*),$$

used previously in the ASRF model (superscript 0 was added here to emphasize the absence of climate impact, which can be obtained by putting either q or α to zero).

A self-consistency condition

If the probability of default without climate risk is $P_D^0 = \Phi(C^*)$ and the probability of default with climate risk is $P_D = \Phi_{q,\hat{\alpha}}(C^*)$, then C^* can be expressed both in terms of climate-exposed and climate risk-free probabilities of default

$$C^* = \Phi^{-1}(P_D^0) = \Phi_{q,\hat{\alpha}}^{-1}(P_D)$$

This means that there is a self-consistency relationship

$$\Phi^{-1}(P_D^0) = \Phi_{q,\hat{\alpha}}^{-1}(P_D) \qquad (6)$$

which links the probabilities and the climate risk parameters.

The parameter α is obligor (asset)-dependent. Parameter q in this model is an objective probability which is supplied by the national Met Office and in, in this binary setup, gives the probability of a weather event above certain ("normal") threshold, thus is not obligor-dependent. Probabilities of default are external in this portfolio model (very much as in the standard Vasicek model which we essentially aim to generalize) and the relation between probability of default in the paper and this external P_D is used to obtain parameter α for each obligor. This is essentially the self-consistency relation between bank's internal P_D and the model P_D.

In particular, the bank's internal estimates of P_D^0 and P_D uniquely define the parameter $\hat{\alpha}$, assuming that a meteorological statistical model externally supplies the parameter q. In short, the non-observable parameter $\hat{\alpha}$ in our generalised Vasicek formulae does not require calibration, this is also true for the non-observable parameterC^*, if the bank's internally modelled P_D^0 and P_D are accepted, together with external statistical estimation for the climate impact q. This observation is essential for the practical implementation of the model.

Probability of default conditional on single economic risk parameter.

We are proposing to introduce an additional stochastic factor in Merton model[22] and due to different (tail) nature of the factor and non-linear payoff of put/calls the average prices for the instruments will be generally different in these models (2-factor vs single factor with changed volatility). The second factor in its "q" nature is the same for all obligors (not idiosyncratic) and is the basis for portfolio (co-dependence) effect.

An expression for the conditional probability of default can be derived as a function of the value of the systematic factor. Once again, this will be a direct generalisation of the corresponding Vasicek's expression.

S being the systemic factor, correlation ρ can be introduced in the absence of climate risk event as follows: $\frac{\sigma_V}{\sqrt{\sigma_V^2+\beta_V^2}} = \sqrt{\rho}$.

One can check that in the Vasicek (2002) model ρ is a correlation between the returns of any two companies' asset values. Using this correlation, it is possible now to calculate the following conditional probability of default for the state with no climate risk-related damage (Vasicek state), recovering the usual Vasicek expression:

$$\mathrm{P}_D(default = 1 \mid S = -y, \xi = \xi_0 = 0) = \mathrm{P}\left(\sqrt{\rho}S + \sqrt{1-\rho}B \leq \Phi^{-1}(\mathrm{P}_D^0) \middle| S = -y, \xi = \xi_0 = 0\right) =$$
$$\mathrm{P}\left(B \leq \frac{1}{\sqrt{1-\rho}}\left(\Phi^{-1}(\mathrm{P}_D^0) - \sqrt{\rho}S\right)\right) = \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(\mathrm{P}_D^0)\right)\right) \quad (7)$$

Analogously, in the presence of climate impact, i.e. $\xi = \xi_1 = 1$, the corresponding conditional probability of default can be calculated as

$$\mathrm{P}_D(default = 1 \mid S = -y, \xi = \xi_1 = 1) = \mathrm{P}\left(B \leq \frac{1}{\sqrt{1-\rho}}\left(\Phi^{-1}(P_D^0) - \sqrt{\rho}S + \hat{\alpha}\right)\right) = \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(\mathrm{P}_D^0) + \hat{\alpha}\right)\right) \quad (8)$$

Symbolically, the above expressions can be presented as

$$\mathrm{P}_D\left(default = 1 \mid S = -y, \xi = \hat{\xi}\right) = \mathrm{P}_R\left(B \leq \frac{\left(\Phi_{q,\hat{\alpha}}^{-1}(\mathrm{P}_D) + \sqrt{\rho}y + \hat{\xi}\hat{\alpha}\right)}{\sqrt{1-\rho}}\right).$$

Combining expressions (7) and (8) together, the probability of default conditional on y in the case of 'binary' climate risk can be obtained:

$$\mathrm{P}_D(default = 1 \mid S = -y) =$$
$$(1-q)\Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(\mathrm{P}_D^0)\right)\right) + q\cdot\Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(\mathrm{P}_D^0)\right) + \frac{\hat{\alpha}}{\sqrt{1-\rho}}\right) = \Phi_{q,\frac{\hat{\alpha}}{\sqrt{1-\rho}}}\left(\frac{\sqrt{\rho}\cdot y + \Phi_{q,\hat{\alpha}}^{-1}(\mathrm{P}_D)}{\sqrt{1-\rho}}\right) \quad (9)$$

[22] In Merton's framework credit corresponds to put option on the firm's assets. Due to the non-linear payout the price of the put option is markedly different for a single-factor distribution with somewhat changed mean and standard deviation and for our two-factor distribution, where the second factor is specifically designed to model the tail event.

Once again, the latter expression is a direct generalisation of Vasicek conditional probability (7). Indeed, formula 9 converges to it if either q or $\hat{\alpha}$ is zero. For a small α , in the first order, Equation (9) is reduced to

$$P_D(default = 1|S = -y) \approx \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(P_D^0)\right)\right) + q\cdot\Phi'\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(P_D^0)\right)\right)\cdot\frac{\hat{\alpha}}{\sqrt{1-\rho}}$$

(10)

Calculating explicitly the last term, one gets the following expression for the conditional probability of default in the first order of q:

$$P_D(default = 1|S = -y) = \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y + \Phi^{-1}(P_D^0)\right)\right) + \frac{q}{\sqrt{2\pi}}\cdot\frac{\hat{\alpha}}{\sqrt{1-\rho}}\cdot e^{-\frac{1}{2}\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot y+\Phi^{-1}(P_D^0)\right)\right)^2}$$

(11)

One of the applications of the above expression, together with the self-consistency Equation (6), is to calculate probability of default conditional on economic factor, which can be used in stress scenarios.

The probability of default for a single borrower conditional on the value of the economic (systematic) factor corresponding to the confidence level Q (CV(Q)), so that it is equal to $\Phi^{-1}(Q)$, can be given by the following expression:

$$C_V(Q) = P_D\left(default = 1|y = \Phi^{-1}(Q)\right) = \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot\Phi^{-1}(Q) + \Phi^{-1}(P_D^0)\right)\right) + \frac{q\cdot\hat{\alpha}}{\sqrt{2\pi(1-\rho)}}\cdot e^{-\frac{1}{2}\left(\frac{1}{\sqrt{1-\rho}}\left(\sqrt{\rho}\cdot\Phi^{-1}(Q)+\Phi^{-1}(P_D^0)\right)\right)^2} \quad (12)$$

The latter equation converges to Vasicek $C_V(Q)$ in the case of zero climate damage, with either q or α equal to zero.

Probability of loss on a portfolio of identical loans.

The proposed credit risk modelling approach derives the cumulative probability of portfolio loss in the generalised ASRF model, closely following the original Vasicek logic (1997,2002). The reader will spot the direct analogy with the original Vasicek calculations as his original derivation and notations are kept as close as possible. This is done to emphasise the minimal changes that have to be made to include the physical climate (weather) -related factor in the ASRF/Basel formula.

The modelling approach starts with highlighting that it is now possible to assign different values of LGD depending on the climate (weather) event occurrence: the authors take LGD for the climate no-event default channel equal to LGD_0 ($\xi = \xi_0 = 0$), while LGD for default in the climate event channel ($\xi = \xi_1 = 1$) is equal to LGD_1. This property allows to model the direct effect of climate (weather) event-related damage on recoverable assets. If, due to the nature of the business, the recoverable assets are not affected by the weather impact, then the two LGD_0 and LGD_1 should be taken equal.

First the case of $\xi = \xi_0 = 0$ (i. e. no climate event channel) can be considered. In this case, the cumulative probability of loss L is given by the following expression:

$$\mathrm{P}_0(Loss < L) = \mathrm{P}_0\left(n < \frac{L}{LGD_0}\right) = \mathrm{P}_0(n < \Theta_0 \cdot N) = \sum_{K=0}^{\Theta_0 N} \mathrm{P}_{k,0}. \tag{13}$$

where

n is the number of borrowers that went bankrupt and Θ_0 is the percentage of defaulted borrowers. Here $\mathrm{P}_{K,0}$ is a probability of K defaulted loans out of N loans in the total portfolio, which can be found using conditional independence of the individual defaults and Formula (7):

$$\mathrm{P}_{K,0} = (1-q)C_N^K \int_{-\infty}^{\infty} d\Phi(y) \left[\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}}\right)\right]^K \left[1-\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}}\right)\right]^{N-K}.$$

In a similar logic, one can find analogue of expression (13) for the cumulative probability of losses in the climate event channel:

$$\mathrm{P}_1(Loss < L) = \mathrm{P}_1\left(n < \frac{L}{LGD_1}\right) = \mathrm{P}_1(n < \Theta_1 \cdot N) = \sum_{K=0}^{\Theta_1 N} \mathrm{P}_{k,1}.$$

In the same way as in the previous no-event case, one can derive $\mathrm{P}_{K,1}$ as follows:

$$\mathrm{P}_{K,1} = qC_N^K \int_{-\infty}^{\infty} d\Phi(y) \left[\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}} + \frac{\hat{\alpha}}{\sqrt{1-\rho}}\right)\right]^K \left[1-\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}} + \frac{\hat{\alpha}}{\sqrt{1-\rho}}\right)\right]^{N-K}$$

We show in Annex 2 that one can use Vasicek's trick to obtain from the two terms above the following expression for the cumulative probability function for portfolio loss *L*:

$$\mathrm{P}(\mathrm{Loss} < L) = q \cdot \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta_1) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}} - \frac{\hat{\alpha}}{\sqrt{\rho}}\right) + (1-q)\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta_0) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)$$

where

$$\Theta_1 = \Theta_0 \frac{LGD_0}{LGD_1}, \qquad \frac{LGD_0}{LGD_1} \leq 1. \tag{14}$$

Formula (14) is one of the main findings of the proposed approach to the credit risk modelling. It shows how by taking into account the self-consistency condition (6), one can calculate the probability of portfolio losses using only the bank's own probabilities of default and external climate statistical estimate *q*.

It is clear that this expression is reduced to the Vasicek function for q = 0 and $\forall\alpha$, and also for α = 0 and $\forall q$, since in this case $LGD_0 = LGD_1$ and $\Theta_1 = \Theta_0$.

Loss given default adjusted for physical risks

The loss given default (LGD) is separate from the Merton P_D. However, the two LGDs (due to credit risk only and adjusted for physical risk) can be connected with the damage (jump in the value of the corporate borrower's assets). One possible approach is to make assumptions about the timing of the jump. For instance, it can be assumed that in case of the climate event, the jump occurs just before the maturity, and V_T is just above L. This can be considered as the most conservative assumption and implies the largest loss of the recoverable assets. Adhering to the spirit of the Basel framework (2017, 2022b) which encourages conservative assumptions for LGD, the modelling will then follow this approach.

Linking the two LGDs:

$L - V_T e^{-\alpha} = LGD_1 \cdot L$ and $L - V_T = LGD_0 \cdot L$.

by solving for $\frac{V_T}{L} = 1 - LGD_0$ and substituting into the first equation:

$1 - e^{-\alpha}(1 - LGD_0) = LGD_1 = (1 - e^{-\alpha}) + LGD_0\, e^{-\alpha}$, i.e.

$$LGD_1 = LGD_0 + (1 - e^{-\alpha})(1 - LGD_0). \tag{15}$$

In this form, Equation (15) allows the interpretation of the term $(1 - e^{-\alpha})(1 - LGD_0)$ as a result of the decreased value of assets that could have been recovered if there were no climate related damages.

For small damage parameter α Equation (15) can be simplified as

$$LGD_1 = LGD_0 + \alpha(1 - LGD_0), \qquad \alpha \ll 1 ,$$

in which case, with the same accuracy, the scaling parameter in Equation (14) is calculated as one minus the ratio of climate-related loss to the pure (economic) credit loss

$$\frac{LGD_0}{LGD_1} = \frac{1}{1+\alpha\frac{1-LGD_0}{LGD_0}} \approx 1 - \alpha\frac{1-LGD_0}{LGD_0} \quad . \tag{16}$$

Expected credit loss modelling

To calculate the Expected Loss, one has to use

$$< EL >= LGD_0 \cdot (1-q)\big(\Phi(C^*)\big) + LGD_1 \cdot q\big(\Phi(C^* + \hat{\alpha})\big),$$

i. e. $< EL >= LGD_0 \cdot (1-q)\Phi\left(\Phi^{-1}_{q,\hat{\alpha}}(P_D)\right) + q \cdot LGD_1 \cdot \left(\Phi_{\hat{\alpha}}\left(\Phi^{-1}_{q,\hat{\alpha}}(P_D)\right)\right) =$

$$= LGD_0 \cdot \Phi_{q,\hat{\alpha}}\left(\Phi^{-1}_{q,\hat{\alpha}}(P_D)\right) + q \cdot (LGD_1 - LGD_0) \cdot \left(\Phi_{\hat{\alpha}}\left(\Phi^{-1}_{q,\hat{\alpha}}(P_D)\right)\right),$$

i. e. $< EL >= LGD_0 \cdot P_D + q \cdot (1 - e^{-\alpha}) \cdot (1 - LGD_0) \cdot \Phi_{\hat{\alpha}}\left(\Phi^{-1}_{q,\hat{\alpha}}(P_D)\right)$,

Here, the first term $LGD_0 \cdot P_D$ is the standard term with the updated probability, while the second term $q \cdot (1 - e^{-\alpha}) \cdot (1 - LGD_0) \cdot \Phi_{\hat{\alpha}}\left(\Phi^{-1}_{q,\hat{\alpha}}(P_D)\right)$ is the exposure to the climate event.

In the lowest order of q and small α, the previous expression takes the form

$$< \mathrm{E}L >= LGD_0 \cdot \mathrm{P}_D + q \cdot (1 - e^{-\alpha}) \cdot (1 - LGD_0) \cdot \mathrm{P}_D$$
$$= LGD_0 \cdot \mathrm{P}_D \cdot \left(1 + (1 - e^{-\alpha}) \cdot q \cdot \frac{1 - LGD_0}{LGD_0}\right) \quad (17)$$

which we will be using below.

Solution for a portfolio credit loss

Let us denote as L_Q^* the level of loss corresponding to the probability of portfolio loss Q in Formula (14).

$\mathrm{P}\left(\mathrm{Loss} < L_Q^*\right) = Q$

$$\mathrm{Q} = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_Q^*}{LGD_0}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right) + q \cdot \left\{\Phi_{-\frac{\hat{\alpha}}{\sqrt{\rho}}}\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_Q^*}{LGD_1}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right) - \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_Q^*}{LGD_0}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)\right\}.$$

We show in Annex 3 that in first order on small parameters one can obtain the following expression for Var level:

$$L_Q^* = LGD_0 \cdot L_{Vasicek} + \alpha \cdot q \cdot (1 - LGD_0) \cdot L_{Vasicek} + \frac{\hat{\alpha}}{\sqrt{2\pi(1-\rho)}} q \cdot e^{-\frac{1}{2}\left(\Phi^{-1}(L_{Vasicek})\right)^2} \cdot LGD_0\,, \quad (18)$$

(18)

with the introduced notation $L_{Vasicek} = \Phi\left(\frac{\sqrt{\rho}\,\Phi^{-1}(Q) + \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{1-\rho}}\right)$.

We can go one step further if we note that $\Phi^{-1}(L_{Vasicek}) = \frac{\sqrt{\rho}\,\Phi^{-1}(Q) + \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{1-\rho}}$ and it is possible to combine the first and third terms of Equation (18) into the term $LGD_0 \cdot C_V(Q)$ according to Equation (12). Furthermore, in the first order on q the second term can be rewritten as $\alpha \cdot q \cdot (1 - LGD_0) \cdot C_V(Q)$ which leads us to the following equation for the Var solution:

$$L_Q^* = LGD_0 \cdot C_V(Q) + \alpha \cdot q \cdot (1 - LGD_0) \cdot C_V(Q) \quad (19)$$

with $C_V(Q)$ defined by Formula (12). Factorizing the right-hand side

$$L_Q^* = C_V(Q) \cdot LGD_0 \cdot \left(1 + \alpha \cdot q \frac{1 - LGD_0}{LGD_0}\right),$$

with the same level of accuracy on α we finally arrived at the expression, which we will substitute in the formula:

$$L_Q^* = C_V(Q) \cdot LGD_0 \cdot \left(1 + (1 - e^{-\alpha}) \cdot q \cdot \frac{1 - LGD_0}{LGD_0}\right). \quad (20)$$

Generalized ASRF formula

We can now substitute expressions (17) and (20) into the definition of the model RWA

Model RWA = Multiples · $\left[L_Q^* - < EL >\right]$

to obtain the following result (generalized Basel formula):

$$\text{Model RWA} = \text{Multiples} \cdot LGD_0 \cdot [C_V(Q) - \mathrm{P}_D\,] \cdot \left(1 + (1 - e^{-\alpha}) \cdot q \cdot \frac{1 - LGD_0}{LGD_0}\right), \tag{21}$$

or, alternatively,

$$\text{Model RWA} = \text{Multiples} \cdot LGD_0 \cdot [C_V(Q) - \mathrm{P}_D\,] \cdot \left(1 + q \cdot \frac{LGD_1 - LGD_0}{LGD_0}\right), \tag{22}$$

In this form, the model RWA has exactly the same form as the model RWA before the inclusion of the climate-related damage (see Equation (3)), with the only difference of the conditional probability of default now accounting for climate risk (see Equation (12)) and the appearance of an additional multiplier $\left(1 + (1 - e^{-\alpha}) \cdot q \cdot \frac{1 - LGD_0}{LGD_0}\right) = (1 + q \cdot \frac{LGD_1 - LGD_0}{LGD_0})$ responsible for reserving against climate-related loss to the recoverable assets at default.

Appendix 1 gives an illustrative example for one of the possible default models and particular area of the US coast. From this example one can see that with the required degree of confidence, to account for climate physical risk-related effect on portfolio of high-quality loans, RWA rises as much as by 20%.

Section 5 – Exploring Further Extensions and Applications of the Enhanced Vasicek Model.

Developing a multi-state climate model

Instead of a binary model for climate impact, a multi-state distribution with $\{q_i, \alpha_i\}_{i=0}^{n-1}$ – probabilities and magnitudes of jumps can be also considered.

In this case, repeating the same steps, one can obtain the following expression for the cumulative probability of portfolio loss:

$$\mathrm{P}(\text{Loss} < L) = \sum_{i=0}^{n-1} q_i\, \Phi\left(\frac{\sqrt{1-\rho}\, \Phi_{-\frac{\hat{\alpha}_i}{\sqrt{\rho}}}\left(\frac{L}{LGD_i}\right) - \Phi^{-1}_{\{\vec{q},\vec{\hat{\alpha}}\}}(\mathrm{P}_D)}{\sqrt{\rho}}\right), \tag{23}$$

where we introduced a vector generalisation of q-deformed normal distribution (5):

$$\Phi_{\{\vec{q},\vec{\hat{\alpha}}\}}(x) = \sum_{i=0}^{n-1} q_i\, \Phi(\mathrm{x} + \hat{\alpha}_i).$$

Equation (23) is a direct generalisation of Formula (14) in the previous section for the case of multi-state climate modelling.

Current paper does not pursuit this formulation, since was aiming to minimize the number of parameters and make the proposed integration of physical risks into the credit risk modelling potentially open for regulation-focused application. However, the multi-state distribution approach can also be taken by banks to build their own internal models.

Potential impact on managing physical risks in credit portfolios

The Vasicek-type credit risk model, enhanced to incorporate physical risk, as described in the paper, retains the key properties of the existing regulatory requirements to credit risk modelling while enabling a more informed measurement and mitigation of physical climate risk factors.

Measuring climate physical risk impact through the lens of credit risk modelling may aid in developing measurement tools for climate-related financial risks using statistical datasets, rather than relying on hypothetical stress scenarios or on policy-driven approaches.

A bank can potentially reduce the assessed credit risk surcharge for physical climate risk by hedging it with derivatives.

Building a Climate damage index swap and hedging climate damage

If banks recognise physical risk or are incentivised by their regulators to recognise it, they would get a strong incentive to offload it from their books, as in practice a bank could hardly manage this type of exogenous risk. The existing financial markets for catastrophic bonds (CAT-bonds) and insurance-linked securities (ILS) are too local and region-specific (US market mainly) and hence small and illiquid for global banks to hedge climate risk even partially.

A type of derivatives designed for large-scale hedging physical risk would probably resemble an existing securitised parametric insurance ILS but with a more standardised and hence liquid underlying, namely a climate damage index.

Such an index calculated on the country-wide level could be traded and thus used to hedge the systematic component of physical climate risk. In order to simplify calculation of losses and speed up settlement, such an index should be parametric, or forward-looking, rather than be based on actuarial losses. This means that the index could be built on the actual values of real estate and other fixed assets, modelled damage functions, and observable weather impacts.

In this regard, the situation is similar to the equity market, where the country's equity index drives the systematic risk of a company, say the S&P 500 for the US or FTSE100 for the UK. The damage index can be traded in the swap format and cleared by central counterparties, utilising the already existing infrastructure currently used to trade and clear credit default swap indices such as CDX IG (for the US) or iTraxx Main (for Europe). The swap would have a floating leg (i. e. the expected damage) and a fixed leg (i. e. its price), with the latter being a clearing price for the traded instrument. Its close analogy will be the market price index, widely available and transparent.

Let us review the formula for model RWA again:

$$\text{Model RWA} = \text{Multiples} \cdot LGD_0 \cdot [C_V(Q) - \mathrm{P}_D\,] \cdot \left(1 + (1 - e^{-\alpha}) \cdot q \cdot \frac{1 - LGD_0}{LGD_0}\right).$$

If one considers pricing swaps on the climate damage index, one can see that in the current model of digital and identical climate risk and universal identical impact, the term $(1 - e^{-\alpha}) \cdot q$ is, in fact, the swap spread (swap). From this point of view the term $\frac{1 - LGD_0}{LGD_0}$ defines the size of the systematic exposure to the swap. In more general cases of non-identical assets in the swap and more complex climate factors, the last multiple should be changed to

$$\left(1 + swap \cdot \beta \cdot \frac{1 - LGD_0}{LGD_0}\right)$$

where

β is the first order sensitivity of damage of recoverable assets to the damage to the swap assets.

The observation about the implicit presence of the swap spread leads to the next question: Whether climate-related damage can be hedged with swaps on the climate damage index?

Formula (21) suggests that in the case of climate damage hedge against ideal counterparty (LGD=0, P_D =0), the probability of default P_D should be put equal to P_D^0, in which case, from Eq(6), $\hat{\alpha} = 0$ and, self-consistently, LGD_0 coincides with LGD_1, returning the model RWA to its "pre-climate" value. Alternatively, one can see the effect of hedging as removing the impact of climate damages on recoverable assets and therefore putting LGD_1 equal to LGD_0, which by the same token will necessitate the equation of P_D and P_D^0 within the model. In either case, Formula (21) will converge to Formula (3). This shows that hedging with climate damage swaps will be an efficient way to manage risk and optimise economic capital.

In this regard, the situation is similar to hedging a credit exposure with a credit default swap, in which case P_D and/or LGD of the borrower is substituted by P_D and/or LGD of the counterparty in the swap. In the case of an ideal counterparty, both of these quantities would be zero, and the model RWA of the hedged loan would be zero, too.

Extension of the model for transition risks

A multi-state generalisation in the previous section has another interesting application. It allows to extend our model to include transition risks as well as physical risks which we have considered so far.

To this end, we can extend the model for the asset process in Section 4 as follows:

$$\widehat{V_t} = V_t\, e^{-\alpha}\, \xi + (1 - \xi)V_t, = V_t(e^{-\alpha}\, \xi + (1 - \xi))$$

with a single (physical risk) jump variable ξ = (0,1) to a process with two independent jumps:

$$\widehat{V_t} = V_t(e^{-\alpha}\, \xi + (1 - \xi)) * (\omega\,(\omega-1)\, e^{-\delta_2}/2 + \omega\,(2-\omega)\, e^{-\delta_1} + (2-\omega)\,(1-\omega)\,/2) \qquad (24)$$

Here the already familiar to us two-state variable ξ related to physical damage with probability q and magnitude e^{-a} < 1 is complemented by a new three-state variable ω=(0, 1, 2) with probabilities $\{p_i\}_{i=0}^2$ and magnitudes of jumps $\{e^{-\delta_i}\}_{i=0}^2$.

The three-state variable ω corresponds to the damage to firm's assets from transition risk, with three states being: ω = 0 no damage (no jump, $e^{-\delta_0}$=1), ω =1 for damage from "orderly transition" (jump down with magnitude $e^{-\delta_1}$) and ω =2 damage from "disorderly transition" (jump down with magnitude $e^{-\delta_2}$).

It is possible to show that in the presence of both physical and transition risks the main results of the proposed model still hold in the first order on small parameters, although the formulas become considerably more cumbersome. In particular, the portfolio *VaR* is equal to the probability of default conditional on a particular value of the systemic parameter, with some multipliers (similar to Equations (20) and (22)). As before, the multipliers reflect climate-related damage to recoverable assets. In the case, when realisation of transition risks does not create "stranded assets", the multiplier will be the same as in Equations (20) and (22).

In the case of a single jump related to physical risk, the self-consistency condition (6) allowed us to solve it for the parameter $\hat{\alpha}$ which is then uniquely defined by bank's internal P_D^0 and P_D, taking into account that the parameter q is externally supplied by a meteorological statistical model. For two jump processes, the situation is more complex. Assuming that for transition risk the magnitudes $\{e^{-\delta_i}\}_{i=0}^{2}$ can be supplied by bank's stress tests for both the "orderly transition" and "disorderly transition" scenarios (several such exercises have been already carried by major banks), one has to obtain model-implied probabilities of jumps $\{p_i\}_{i=0}^{2}$. This requires two additional self-consistency equations, similar to Equation (6), which are explicitly linked to transition risk. The extended system of self-consistency conditions can be written as:

$$\Phi^{-1}(\mathrm{P}_D^0) = \Phi_{q,\hat{a}}^{-1}\big(\mathrm{P}_{D,physical}\big)\,,$$
$$\Phi^{-1}(\mathrm{P}_D^0) = \Phi_{p,\hat{\delta}}^{-1}\big(\mathrm{P}_{D,transition}\big)\,,$$
$$\Phi^{-1}(\mathrm{P}_D^0) = \Phi_{(q,p)(\hat{a},\hat{\delta}),}^{-1}(\mathrm{P}_D)\,, \tag{25}$$

where

P_D^0 is the bank's internally modelled probability of borrower's default if no climate risk, be it transition or physical, are taken into account,

$\mathrm{P}_{D,physical}$ is the bank's internally modelled probability of borrower's default if pure physical risk is taken into account,

$\mathrm{P}_{D,transition}$ is the bank's internally modelled probability of borrower's default if pure transition risk is taken into account,

and, finally, P_D is the bank's internally modelled probability of borrower's default if both physical transition risks are included.

All these bank's internally modelled probabilities require, in fact, a single model for joint risks, PD, where either physical risk parameters or transition risk parameters can be put to zero. Functions $\Phi_{p,\hat{\delta}}$ and $\Phi_{(q,p)(\hat{a},\hat{\delta})}$ are the corresponding generalisations of the "q-deformer normal distribution" $\Phi_{q,\hat{a}}$ defined in Equation (5).

The asset process (24) together with self-consistency equations (24) fully define an explicitly solvable extension of the original Vasicek model for the credit portfolio loss functions when both physical and transition risks are included and calibrated to the bank's internal models for individual probabilities of default. One example of such an internal model can be Moody's Climate-Adjusted EDF (CEDF) model outlined above in Section 2. Other models, which take into account explicit weather stochastic modelling and geo-positioning of the assets to arrive at climate-adjusted probabilities of default, are also commercially available.

Section 6 – Concluding remarks

This research in credit risk modelling can be concluded with several observations regarding the practical use of Formula (21).

To calculate the model RWA according to the Generalised ASRF Equation (21), banks need to have:

1. A current internal credit risk model which estimates the probability of default P_D^0 (not explicitly accounting for risk of climate-related damage).
2. An internal (or a third-party) model that will produce P_D from P_D^0, explicitly accounting for the climate-related damage impact on the probability of default.

3. An external statistical meteorological model of climate impact, which produces the probability of a climate-related event in the Generalised ASRF model, q.
4. A current internal model to estimate loss given default not adjusted for physical risk, LGD_0.
5. A conservative estimate of loss given default in the case of a climate-related damage, LGD_1.

Indeed, P_D and P_D^0will define, together with $\hat{\alpha}$ found from Equation (6), the first bracket in expression (21), while LGD_0 and LGD_1, together with the external statistic q, will define the second bracket.

This poses the question of building a bank's internal model for climate-related correction to the internal probability of default and LGD. This model needs to take into account the type of borrower's assets, their geo-location, climate exposure, statistical description of weather events and damage functions.

While banks can develop this expertise themselves, a RegTech solution which would allow them to outsource this work to trusted service providers whose solutions can be centrally audited, including by regulators, may be beneficial.

Finally, the Vasicek-type credit risk model enhanced to incorporate physical risks and described by the paper may be of specific interest for both banks and regulators, as it preserves the important properties of existing regulation while allowing for an informed mitigation of climate risk factor in credit risk. This innovative credit-risk modelling supports the development of more resilient financial systems that can adapt to evolving environmental challenges.

Annex 1 Illustrative example

Let us assume that we add a loan to an investment grade company secured by its commercial real estate located in Mobile, Alabama.

Value of *q*

The analysis from S&P[23] suggests that the probability of a major hurricane (Grade 3 or higher) for 2025 is around 3% (see Fig. 1), which is 76% higher than its long-term average (1.7%). We take q equal to 3%.

We also will demonstrate below the effect for q equal to 4.8%, which corresponds to 95% confidence level estimate for probability of hurricane of Grade 3 in 2025 (Fig. 3).

[23] https://www.spglobal.com/esg/insights/featured/special-editorial/an-elevated-2025-hurricane-season

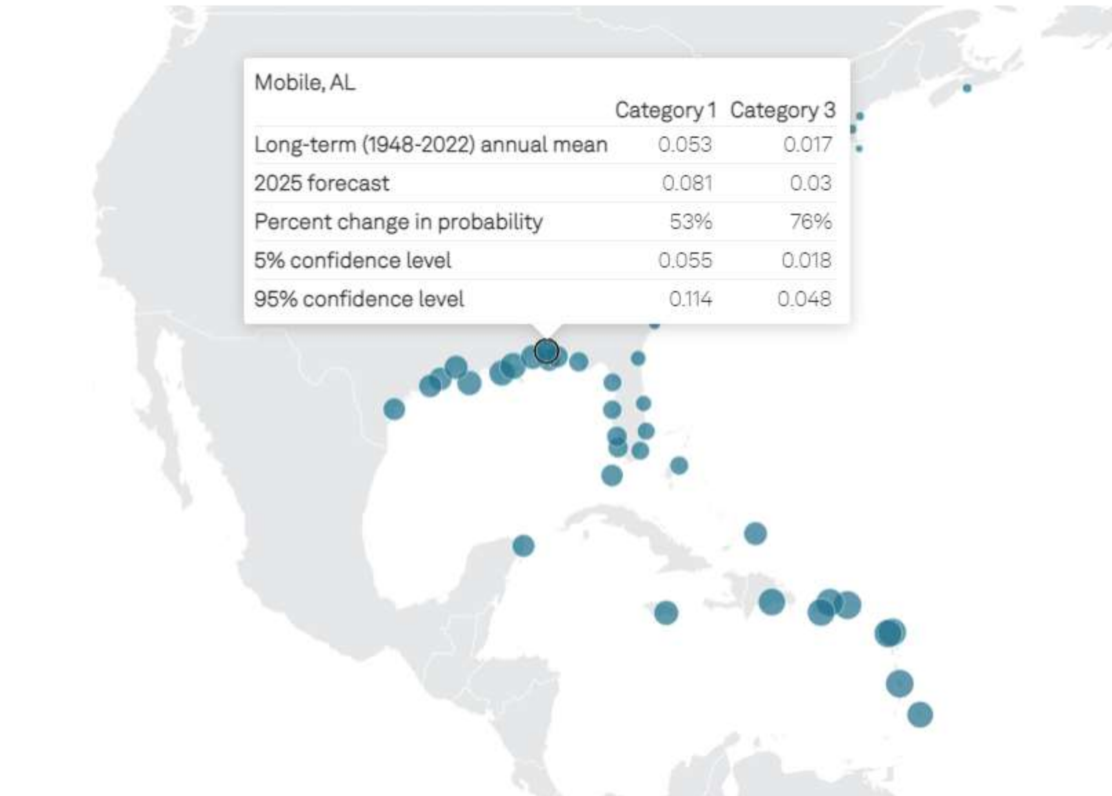
Figure 3

Hurricane probabilities are higher in 2025 throughout the western Atlantic

Probability of a tropical cyclone passing within 50 km of a sample of coastal locations, ranked by highest Category 1 or higher 2025 forecast

To switch between Category 1 and Category 3 views, please click the buttons below; Hover over/click dots for details

Category 1 | Category 3



Probabilities of defaults from internal models

Here, we (acting "as a bank") selected a multi-agent modelling methodology rather than a pure stochastic Merton-style model to mimic a bank's own internal model which is not necessarily will be Merton-type and will include specifics of the firm in question.

Recent research by De Spiegeleera et al. (2024)[24] examined an increase in the probability of defaults due to climate change from a rather interesting perspective of impact on supply chain using multi-agent modelling.

The research combined chain, finance, and financial models to estimate an impact on firm production and the corresponding increase in default probability.

[24] De Spiegeleera et al. (2025).

This research found by simulations that *"where each disaster type's frequency was doubled keeping the frequencies and intensities of other disaster types constant the disaster type with the greatest impact on the average number of defaulted firms is the storm type (+16.1%), followed by the flood (+13.4%), drought (+7.9%), and wildfire (+5.6%) disaster types"*.

This allows us to estimate a relative increase of the probability of default in case of the hurricane danger prevailing on this territory.

Taking into account:

1. The relative increase of the forecasted probability frequency q by 76%,
2. The long-term frequency of 1.7% corresponding to the probability of default which we called the "pre-climate" P_D^0, as it does not include acute physical risk ,
3. A linear dependence of the increase of the probability of default on an increase of the hurricane frequency,

we obtain the following predicted increase of P_D:

$$P_D / P_D^0 = 1 + 16.1\% \cdot 0.76 = 1.1224.$$

This is the percentage increase of the probability of default due to included acute climate risk which we are going to use in our formula for unexpected credit loss.

Let us now assume that the company in question is a BBB-rated company, and the bank's internal model generates the one year probability of default (P_D^0) of 0.3%. This is line with the 1981-2024 average 7-year global cumulative default rate for corporates of 2.15% for BBB-rated companies[25].

From the formula above, we obtain the "physical risk"-adjusted P_D =0.337%.

Values of LGD_0

The last required component is LGD_0. The long-term average loan recovery rates reported by Loan Syndications and Trading Association (Credit Metrics) [26] are just above 80%

Recoveries are typically higher for secured loans to high-quality companies, so we assume here the loan recovery rate of 90% for our company, i.e. LGD_0 = 10%, which is in line with the Basel III minimum requirements under the advanced IRB approach (BCBS 2017, para 85).

These parameters allow us now to calculate the model unexpected loss and the relative model RWA correction.

Calculations

[25] S&P Global Ratings (2025).

[26] Coffey (2021).

Formula (2) for the correlation from the Basel IRB approach ρ gives the value of 22.3%.

The self-consistency condition (6) allows us to find the implied value of parameter α. In our case, the solution is $\hat{\alpha}$= 0.58.

$C_V(Q)$ without physical risk is equal 0.0720, $C_V(Q)$ with climate correction is equal to 0.0747. This gives the difference in the conditional and unconditional probabilities ($C_V(Q)$-P_D) without climate risk correction equal to 0.0690 and, accounting for climate, 0.0714, i.e. the increase of approximately 3.4%.

The model-based calculation of LGD_1 requires knowledge of parameter α rather than $\hat{\alpha}$, which we solved for above. The parameter α is equal to $\hat{\alpha}$ multiplied by of the firm's asset value volatility.

This volatility is equal to the volatility of the firm's equity divided by the sensitivity of the equity to the firm's asset value (i.e. delta of the call option). Since the probability of default is so tiny, it is safe to take the sensitivity (delta) equal to 1 since the (equity) call option of the assets is deeply in the money. The typical value of forward-looking (implied) volatility for high quality US stocks can be taken as 30% which leads us to the value of asset volatility equal to 0.3. This results in the value of alpha equal to $0.3 \cdot \hat{\alpha}$ = 17.4% which corresponds to $e^{-\alpha}$=0.84 (16% reduction in asset value due to the climate event damage). This leads to the value of LGD_1 equal to 24.8%, i.e. the increase in loss given default by 14.8%.

Finally, unexpected loss without our correction related to physical risk (LGD_0=10%) is equal 0.0069 while with the correction and calculated LGD_1 (24.8%) the unexpected loss will be equal to 0.0744, i.e. the **relative increase of 7.9%**. This also mean that the Model RWA, when physical risk-related corrections are included, increases by around 7.9% compared to the original calculations made without add-ons for specific physical risk.

In the above calculation, we utilised the model-calculated LGD_1. In practice, similar to LGD_0 which is taken from outside the probabilistic Merton model, LGD_1 also can be taken, conservatively, from outside the model. For example, if LGD_1 is expected to be 40% (which corresponds to approximately 33% damage to the firm's assets), then the relative increase of the unexpected loss and the Model RWA will rise to **12.7%**.

The case of 95% confidence level *q*. We now repeat the calculation, but will take this time q equal to 4.8%, which corresponds to 95% confidence level estimate for the probability of hurricane of Grade 3 in 2025 (Fig. 1).

In this case, P_D is found from our "16.1%" rule as 0.0039 (which is probably an underestimation due to nonlinear character of the damage impact on the P_D for higher frequencies) and $\hat{\alpha}$ is equal to 0.72. In this case, the difference in conditional and unconditional probabilities ($C_V(Q)$-P_D) without correction for physical risk equal to 0.0690 and, accounting for physical climate risk, 0.0735, i.e. the increase of the approximately 6.5%. The value of alpha equal to $0.3 \cdot \hat{\alpha}$ = 21.6% which corresponds to $e^{-\alpha}$=0.805 (i. e. 19.5% reduction in asset value due to the climate event damage). This leads to the value of LGD_1 equal to 27.5%, i.e. the increase in model-calculated loss given default by 17.5%.

Finally, the unexpected loss without correction for physical risk (LGD_0=10%) remains equal to 0.0069 while with climate correction and calculated LGD_1 (27.5%) the unexpected loss will be equal to 0.0797, i.e. relative increase of 15.5%. This also means that the Model RWA, when physical risk -related adjustments are included, increases by around 15.5% compared to the original calculations. This number rises even further to nearly 22% when the Model LGD_1 is substituted by the external *LGD* value of 40%.

In summary, we can see that in this real-life example with the required degree of confidence, to account for climate physical risk-related effect on portfolio of high-quality loans, RWA rises as much as by 20%.

It is important to highlight that the resulting capital effect is not for some time in a distant and uncertain future, for which weather forecasts are derived from one of the IPCC scenarios. Instead, it is specifically calculated for the year 2025, underscoring the timely relevance of the proposed methodology in incorporating climate-related physical risks into credit risk modelling.

Annex 2 Derivation of cumulative probability function for portfolio loss

First = the case of ξ = ξ_0 = 0 (i. e. no climate event channel) can be considered. In this case, the cumulative probability of loss L is given by the following expression:

$$\mathrm{P}_0(Loss < L) = \mathrm{P}_0\left(n < \frac{L}{LGD_0}\right) = \mathrm{P}_0(n < \Theta_0 \cdot N) = \sum_{K=0}^{\Theta_0 N} \mathrm{P}_{k,0.}$$

where

n is the number of borrowers that went bankrupt;

Θ_0 is the percentage of defaulted borrowers

Here $\mathrm{P}_{k,0.}$ is a probability of K defaulted loans out of N loans in the total portfolio, which can be found using conditional independence of the individual defaults and Formula (7):

$$\mathrm{P}_{K,0} = (1-q)C_N^K \int_{-\infty}^{\infty} d\Phi(y) \left[\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}}\right)\right]^K \left[1-\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}}\right)\right]^{N-K}.$$

To simplify the above integral one can introduce a new variable:

$$s = \Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}}\right),$$

so that both the original variable y and its functional value $\Phi(y)$ can be easily found:

$$y = \frac{\sqrt{1-\rho}\,\Phi^{-1}(s)-\Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}} \quad \text{and} \quad \Phi(y) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(s)-\Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right) = W(s) \ .$$

These notations allow to rewrite the expression for $\mathrm{P}_{k,0.}$ in a simplified form

$$\mathrm{P}_{K,0} = (1-q)C_N^K \int_0^1 dW(s) s^K (1-s)^{N-K}.$$

Substituting the above expression into the sum (13), the cumulative probability of losses in the no-climate-event channel can be obtained as follows:

$$\mathrm{P}_0(Loss < L) = (1-q)\int_0^1 dW(s) \sum_{K=0}^{\Theta_0 N} s^K (1-s)^{N-K} C_N^K = (1-q)\int_0^1 dW(s)\theta(\Theta_0 - s) = (1-q)W(\Theta_0).$$

Here, the Vasicek's (1987) original observation about the Heavyside Theta (step) function $\theta(\bullet)$ convergence was used:

$$\lim_{N\to 0} \sum_{K=0}^{\Theta_0 N} s^K (1-s)^{N-K} C_N^K = \begin{cases} 0, s > \Theta_0 \\ 1, s < \Theta_0 \end{cases} = \theta(\Theta_0 - s).$$

Similarly, one can find the cumulative probability of losses in the climate event channel:

$$\mathrm{P}_1(Loss < L) = \mathrm{P}_1\left(n < \frac{L}{LGD_1}\right) = \mathrm{P}_1(n < \Theta_1 \cdot N) = \sum_{K=0}^{\Theta_1 N} \mathrm{P}_{k,1.}$$

In the same way as in the previous no-event case, one can derive $\mathrm{P}_{K,1}$ as follows:

$$\mathrm{P}_{K,1} = qC_N^K \int_{-\infty}^{\infty} d\Phi(y) \left[\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}} + \frac{\hat{a}}{\sqrt{1-\rho}}\right)\right]^K \left[1-\Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}} + \frac{\hat{a}}{\sqrt{1-\rho}}\right)\right]^{N-K}$$

and the total cumulative probability

$$\mathrm{P}_1 = q\int_0^1 dW_1(s)\sum_{K=0}^{\Theta_0 N} s^K(1-s)^{N-K}C_N^K = q\int_0^1 dW_1(s)\Theta(\Theta_1 - s) = qW_1(\Theta_1).$$

The only difference in the final expression is the substitution of function W_1 for W which is defined below:

$$s = \Phi\left(\frac{\Phi^{-1}(\mathrm{P}_D^0)+\sqrt{\rho}\cdot y}{\sqrt{1-\rho}} + \frac{\hat{\alpha}}{\sqrt{1-\rho}}\right) \Rightarrow y = \left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(s) - \frac{\hat{\alpha}}{\sqrt{1-\rho}}\sqrt{1-\rho} - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right),$$

hence

$$W_1(s) = \Phi(y) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(s) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}} - \frac{\hat{\alpha}}{\sqrt{\rho}}\right), \text{ i.e.}$$

$$\mathrm{P}_1 = q\cdot\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta_1) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}} - \frac{\hat{\alpha}}{\sqrt{\rho}}\right).$$

By combining together P_0 and P_1, one can finally obtain the cumulative probability function for portfolio loss L :

$$\mathrm{P}(\mathrm{Loss} < L) = q\cdot\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta_1) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}} - \frac{\hat{\alpha}}{\sqrt{\rho}}\right) + (1-q)\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta_0) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)$$

where

$$\Theta_1 = \Theta_0\frac{LGD_0}{LGD_1}, \qquad \frac{LGD_0}{LGD_1} \le 1.$$

Annex 3 Derivation of solution for a Value at Risk

Let us denote as L_Q^* the level of loss corresponding to the probability of portfolio loss Q in Formula (14).

$$\mathrm{P}\left(\mathrm{Loss} < L_Q^*\right) = Q$$

$$\mathrm{Q} = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_Q^*}{LGD_0}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right) + q\cdot\left\{\Phi_{-\frac{\hat{\alpha}}{\sqrt{\rho}}}\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_Q^*}{LGD_1}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right) - \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_Q^*}{LGD_0}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)\right\}.$$

Having the probability of a climate-related disaster event q as a true small parameter, let us solve this equation for L_Q^* using the perturbation theory in the series of q:

$$L_Q^* = L_{Q,0}^* + qL_{Q,1}^* + q^2L_{Q,2}^* + q^3L_{Q,3}^* + \cdots \quad .$$

Then, in the order of q^0, $Q = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{L_{Q,0}^*}{LGD_0}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)$, which gives

$$L_{Q,0}^* = LGD_0 \cdot L_{Vasicek},$$

where we introduce the notation for the well-known Vasicek solution

$$L_{Vasicek} = \Phi\left(\frac{\sqrt{\rho}\,\Phi^{-1}(Q) + \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{1-\rho}}\right).$$

Then, in the order of q^1, Equation (18) results in the following equation

$$0 = \frac{\partial}{\partial\Theta}\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)\Bigg|_{\Theta=L_{Vasicek}} \cdot q \cdot \frac{L_{Q,1}^*}{LGD_0} + q\left\{\Phi_{-\frac{\hat{\alpha}}{\sqrt{\rho}}}\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(L_{Vasicek}\frac{LGD_0}{LGD_1}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right) - Q\right\}$$

which can be solved to find $L_{Q,1}^*$:

$$L_{Q,1}^* = LGD_0\frac{Q - \Phi_{-\frac{\hat{\alpha}}{\sqrt{\rho}}}\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(L_{Vasicek}\frac{LGD_0}{LGD_1}\right) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)}{\frac{\partial}{\partial\Theta}\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)\Big|_{\Theta=L_{Vasicek}}}.$$

Again, for α = 0 the correction turns to zero: $L_{Q,1}^* = 0$. For small α ≪ 1, we can use approximation (16) to simplify the above solution further:

$$L_{Q,1}^* = \frac{\hat{\alpha}}{\sqrt{\rho}}\frac{\Phi'\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(L_{Vasicek}) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)}{\frac{\partial}{\partial\Theta}\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta) - \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)\Big|_{\Theta=L_{Vasicek}}} \cdot LGD_0 + \alpha \cdot L_{Vasicek} \cdot \frac{1-LGD_0}{LDG_0} \cdot LGD_0.$$

Taking into account the explicit form of the probability density for Vasicek distribution

$$\frac{\partial}{\partial\Theta}\,\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\Theta)-\ \Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{\rho}}\right)=\sqrt{\frac{1-\rho}{\rho}}\,e^{-\frac{1}{2\rho}\left(\sqrt{1-\rho}\,\Phi^{-1}(\Theta)-\Phi^{-1}(\mathrm{P}_D^0)\right)^2}\cdot\ e^{\frac{1}{2}\left(\Phi^{-1}(\Theta)\right)^2}$$

we finally arrive at the following expression for $L^*_{Q,1}$:

$$L^*_{Q,1}=\alpha(1-LGD_0)\,L_{Vasicek}+\frac{\hat{\alpha}}{\sqrt{2\pi(1-\rho)}}\,e^{-\frac{1}{2}\left(\Phi^{-1}(L_{Vasicek})\right)^2}.$$

This results in the Eq (18) which we were aiming to derive:

$$L^*_Q=LGD_0\cdot L_{Vasicek}+\alpha\cdot q\cdot(1-LGD_0)\cdot L_{Vasicek}+\frac{\hat{\alpha}}{\sqrt{2\pi(1-\rho)}}\,q\cdot e^{-\frac{1}{2}\left(\Phi^{-1}(L_{Vasicek})\right)^2}\cdot LGD_0\ ,$$

where, again, we use the notation $L_{Vasicek}=\Phi\left(\frac{\sqrt{\rho}\,\Phi^{-1}(Q)+\,\Phi^{-1}(\mathrm{P}_D^0)}{\sqrt{1-\rho}}\right)$.

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